Chapter 9 Trigonometry

1. (a) Solve $tan(\alpha + 45°) = -\frac{1}{\sqrt{2}}$ for $0° \leq \alpha \leq 360°$.

[3]

$\alpha + 45° = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $\quad 45° \leq \alpha + 45 \leq 405°$

$= 35.3°$

S	A
T	C

For negative;

$\alpha + 45° = 180° - 35.3°, 360 - 35.3°$

$\alpha + 45° = 144.7°, 324.7$

$\alpha = 99.7°, 279.7°$

(b)(i) Show that $\frac{1}{sin\theta - 1} - \frac{1}{sin\theta + 1} = asec^{2}\theta$, where *a* is a constant to be found.

[3]

$$L.H.S = \frac{\cancel{\sin\theta}+1-\cancel{\sin\theta}+1}{\sin^2\theta - 1}$$

$$= \frac{2}{-(1-\sin^2\theta)} = \frac{-2}{\cos^2\theta}$$

$$= -2\sec^2\theta$$

$$a = -2$$

(ii) Hence solve $\frac{1}{sin3\phi-1} - \frac{1}{sin3\phi+1} = -8$ for $-\frac{\pi}{3} \leq \phi \leq \frac{\pi}{3}$ radians.

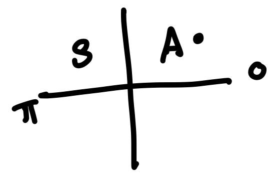
[5]

$\cancel{-}2\sec^2 3\phi = \cancel{-}8$ $\qquad -\pi \leq 3\phi \leq \pi$

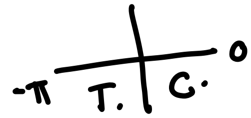
$\sec^2 3\phi = 4$

$\cos^2 3\phi = \frac{1}{4}$

$\cos 3\phi = \pm\frac{1}{2}$



$\cos 3\phi = \frac{1}{2}$ or $\cos 3\phi = -\frac{1}{2}$



$3\phi = \cos^{-1}\left(\frac{1}{2}\right)$

$3\phi = \frac{\pi}{3}, -\frac{\pi}{3}$

$\phi = \frac{\pi}{9}, -\frac{\pi}{9}$

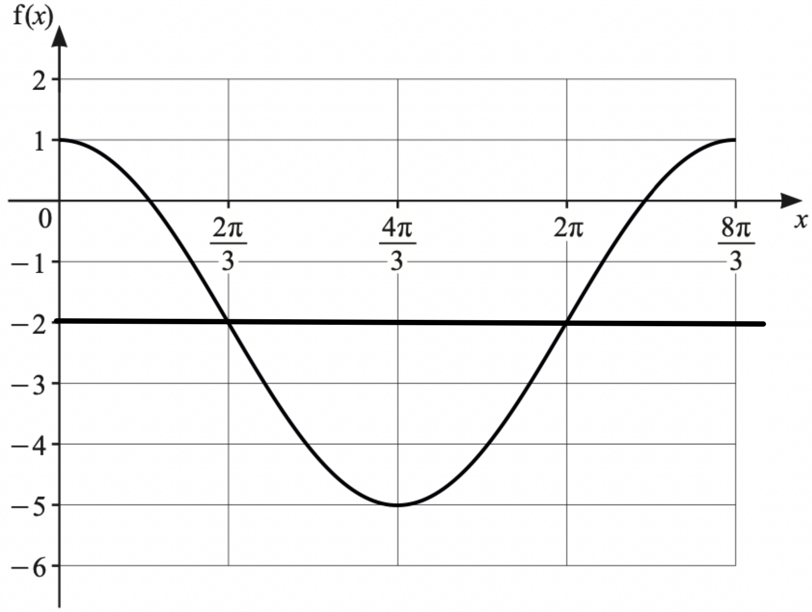
For negative,

$3\phi = \pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$

$= \frac{2\pi}{3}, -\frac{2\pi}{3}$

$\phi = \frac{2\pi}{9}, -\frac{2\pi}{9}$

2.



The diagram shows the graph of $f(x) = acosbx + c$ for $0 \leq x \leq \frac{8\pi}{3}$ radians.

a. Explain why f is a function.

many to one mapping [1]

b. Write down the range of f.

$-5 \leq y \leq 1$ [1]

c. Find the value of each of the constants *a*, *b* and *c*.

$C = -2$ [4]

$a = 3$

$b = \frac{3}{4}$

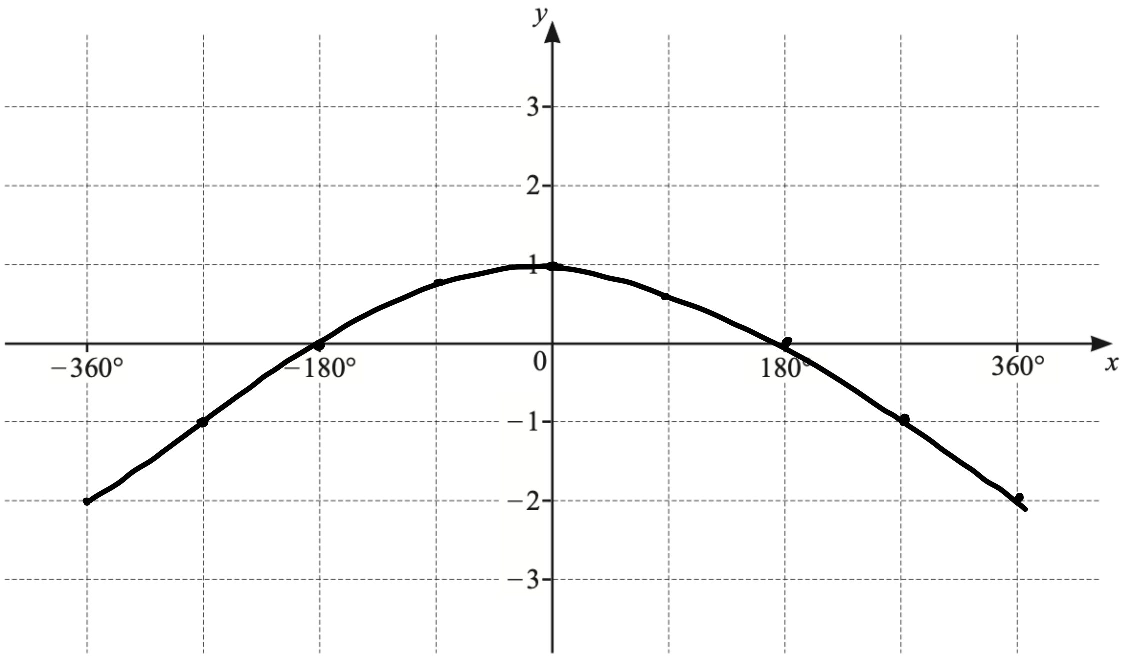
3. (a) Write down the period of $2cos\frac{x}{3} - 1$.

1080°

[1]

(b) On the axes below, sketch the graph of $y = 2cos\frac{x}{3} - 1$. for -360°≤ x ≤360°

[3]



4. (a)(i) Show that $\frac{1}{sec\theta-1} - \frac{1}{sec\theta+1} = 2cot^2\theta$.

[3]

$$L.H.S = \frac{\cancel{\sec\theta}+1-\cancel{\sec\theta}+1}{\sec^2\theta-1}$$

$$= \frac{2}{\tan^2\theta} = 2\cot^2\theta$$

$$= R.H.S$$

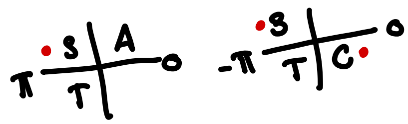
5. (a) Solve $tan\,3x = -1$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ radians, giving your answers in terms of π.

[4]

$-\frac{3\pi}{2} \leqslant 3x \leqslant \frac{3\pi}{2}$

$3x = \tan^{-1}(1)$

$3x = \frac{\pi}{4}$



For negative,

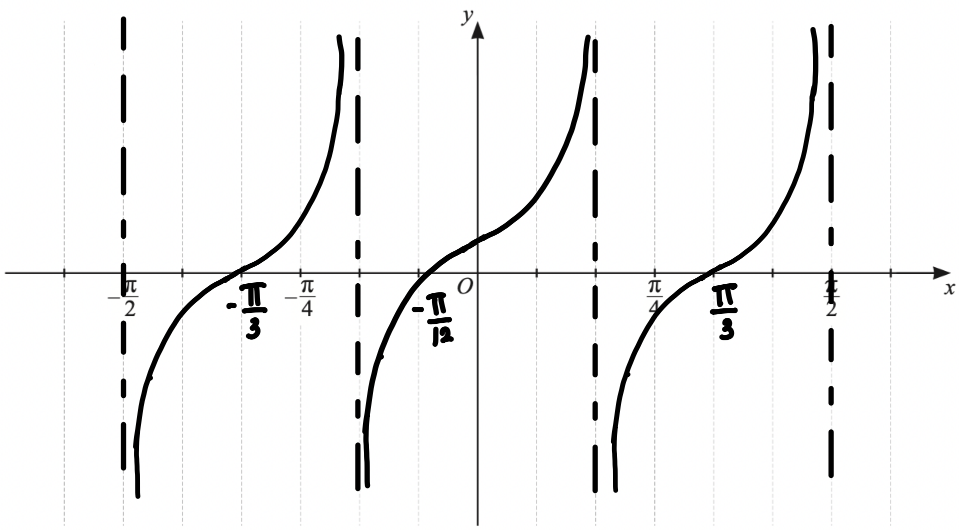
$3x = \pi - \frac{\pi}{4}, -\frac{\pi}{4}, -\pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}, -\frac{\pi}{4}, -\frac{5\pi}{4}$

$x = \frac{\pi}{4}, -\frac{\pi}{12}, -\frac{5\pi}{12}$

(b) Use your answers to **part (a)** to sketch the graph of $y = 4tan3x + 4$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ radians on the axes below. Show the coordinates of the points where the curve meets the axes.

[3]



6. (a) Solve $3cot^2 x - 14\, cosec\, x - 2 = 0$ for 0°< x <360°.

[5]

$3(cosec^2 x - 1) - 14\, cosec\, x - 2 = 0$

$3cosec^2 x - 3 - 14cosec\, x - 2 = 0$

$3cosec^2 x - 14\, cosec\, x - 5 = 0$

$(3cosec\, x + 1)(cosec\, x - 5) = 0$

$cosec\, x = -\frac{1}{3}$ or $cosec\, x = 5$

$\sin x = -3$ (reject)

$\sin x = \frac{1}{5}$

$x = \sin^{-1}\left(\frac{1}{5}\right)$

$= 11.5°, 180 - 11.5$

$= 11.5°, 168.5°$

(b) Show that $\frac{sin^4 y - cos^4 y}{cot\, y} = tany - 2\, cosy\, siny$.

[4]

$L.H.S = \frac{(sin^2 y - cos^2 y)\cancel{(sin^2 y + cos^2 y)}^{1}}{coty}$

$= \frac{sin^2 y}{coty} - \frac{cos^2 y}{coty}$

$= sin^2 y \times \frac{siny}{cosy} - cos^{\cancel{2}} y \times \frac{siny}{\cancel{cosy}}$

$= tany(1 - cos^2 y) - cosy\, siny$

$= tany - cosy\, siny - cosy\, siny$

$= tany - 2cosy\, siny$ (shown)

7. (a)The curve $y = a \sin bx + c$ has a period of 180°, an amplitude of 20 and passes through the point (90°, -3). Find the value of each of the constants a, b and c.

[3]

$b = \frac{360}{180} = 2$

$a = 20$

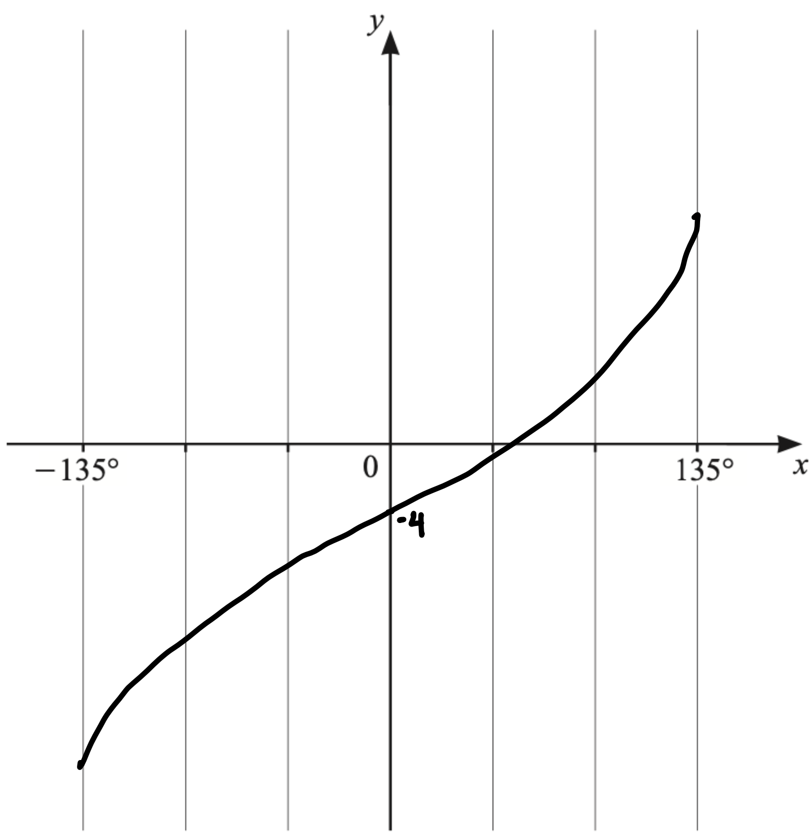
$y = 20 \sin 2x + c$

$-3 = 20 \sin 180° + c$

$-3 = c$

(b) The function g is defined, for -135°≤ x ≤135°, by $g(x) = 3tan\frac{x}{2} - 4$.Sketch the graph of $y = g(x)$ on the axes below, stating the coordinates of the point where the graph crosses the y-axis.

[2]



8. Solve the equation.

a. $5sec^2A + 14tanA - 8 = 0$ for $0° \le A \le 180°$, [4]

$5(\tan^2 A + 1) + 14\tan A - 8 = 0$

$5\tan^2 A + 5 + 14\tan A - 8 = 0$

$5\tan^2 A + 14\tan A - 3 = 0$

$(5\tan A - 1)(\tan A + 3) = 0$

$\tan A = \frac{1}{5}$ or $\tan A = -3$

$A = \tan^{-1}\left(\frac{1}{5}\right)$

$= 11.3$

$A = \tan^{-1}(3)$

$= 71.6°$

For negative,

$A = 108.4$

b. $5sin(4B - \frac{\pi}{8}) + 2 = 0$ for $-\frac{\pi}{4} \le B \le \frac{\pi}{4}$ radians. [4]

$\sin(4B - \frac{\pi}{8}) = -\frac{2}{5}$

$4B - \frac{\pi}{8} = \sin^{-1}\left(\frac{2}{5}\right)$

$= 0.412$

For negative,

$4B - \frac{\pi}{8} = -0.412, -\pi + 0.412$

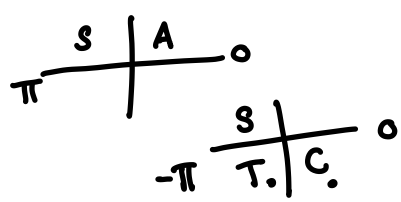
$= -0.412, -2.73$

$4B = -0.0193, -2.34$

$B = -0.004825, -0.585$

$-\pi \le 4B \le \pi$

$-\frac{9\pi}{8} \le 4B - \frac{\pi}{8} \le \frac{7\pi}{8}$



9. (a) Write down the amplitude of $1 + 4\,cos(\frac{x}{3})$.

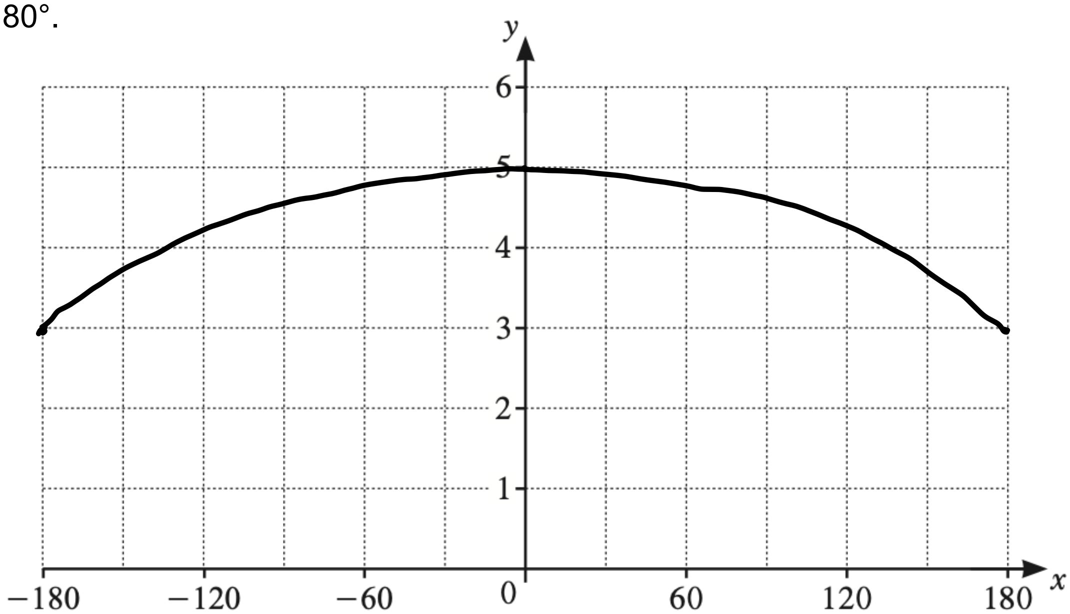
4 [1]

(b) Write down the period of $1 + 4\,cos(\frac{x}{3})$.

1080 [1]

(c) On the axes below, sketch the graph of $y = 1 + 4\,cos(\frac{x}{3})$ for -180° $\leq x \leq$ 180°.

[3]



10. (a)(i) Show that $\frac{1}{(1+cosec\,\theta)(sin\theta - sin^2\theta)} = sec^2\theta$.

[4]

$$L.H.S = \frac{1}{(1+\frac{1}{\sin\theta})\sin\theta(1-\sin\theta)}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cancel{1}-\sin\theta+\frac{1}{\sin\theta}\cancel{-1}}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\frac{1-\sin^2\theta}{\sin\theta}}$$

$$= \frac{1}{\sin\theta} \times \frac{\sin\theta}{1-\sin^2\theta} = \frac{1}{1-\sin^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

(shown)

(ii) Hence solve $(1 + cosec\,\theta)(sin\theta - sin^2\theta) = \frac{3}{4}$ for $-180° \leq \theta \leq 180°$.

[4]

$$\frac{1}{\sec^2\theta} = \frac{3}{4}$$

$$\sec^2\theta = \frac{4}{3}$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

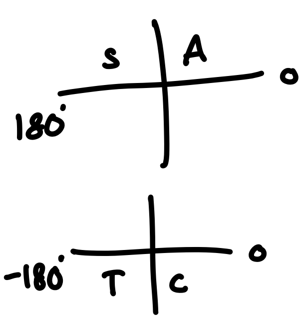
$\cos\theta = \frac{\sqrt{3}}{2}$ or $\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 30°, -30°$

For negative,

$\theta = 150, -150$



(b) Solve $sin(3\phi + \frac{2\pi}{3}) = cos(3\phi + \frac{2\pi}{3})$ for $0 \leq \phi \leq \frac{2\pi}{3}$ radians, giving your answers in terms of π.

[4]

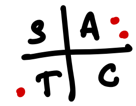
$0 \leqslant 3\phi \leqslant 2\pi$

$\frac{2\pi}{3} \leqslant 3\phi + \frac{2\pi}{3} \leqslant \frac{8\pi}{3}$

$\tan(3\phi + \frac{2\pi}{3}) = 1$

$3\phi + \frac{2\pi}{3} = \tan^{-1}(1)$

$= \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$



$3\phi = -\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}$

$\phi = -\frac{5\pi}{36}, \frac{7\pi}{36}, \frac{19\pi}{36}$

(reject)

11. (a) Write down the amplitude of $2cos\,\frac{x}{3} - 1$.

2

[1]

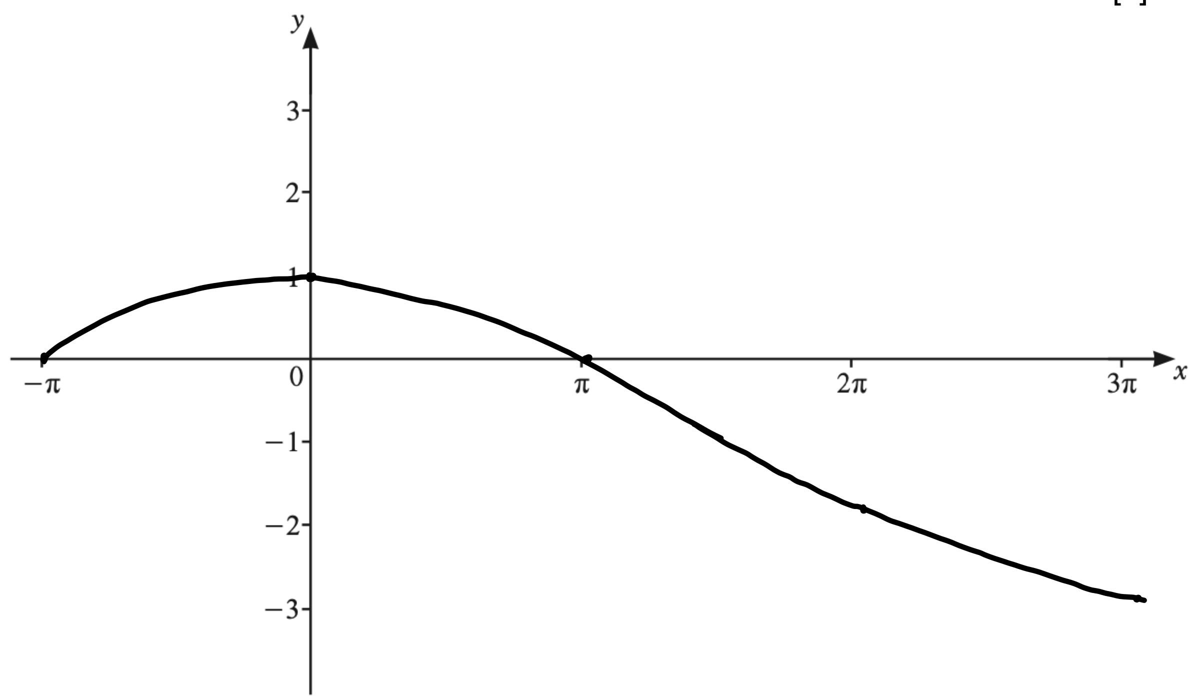
(b) Write down the period of $2cos\,\frac{x}{3} - 1$.

1080

[1]

(c) On the axes below, sketch the graph of $y = 2cos\,\frac{x}{3} - 1$ for $-\pi \leq x \leq 3\pi$ radians.

[3]



12. (a) Given that $2\,cos x = 3\,tan x$, show that $2sin^2 x + 3sin\,x - 2 = 0$. [3]

$$2\cos x = 3\,\frac{\sin x}{\cos x}$$

$$2\cos^2 x = 3\sin x$$

$$2(1-\sin^2 x) = 3\sin x$$

$$2 - 2\sin^2 x - 3\sin x = 0$$

$$2\sin^2 x + 3\sin x - 2 = 0$$

(shown)

(b) Hence solve $2\,cos\,(2\alpha + \frac{\pi}{4}) = 3\,tan\,(2\alpha + \frac{\pi}{4})$ for $0 < \alpha < \pi$ radians, giving your answers in terms of π. [4]

$$0 < 2\alpha < 2\pi$$

$$\frac{\pi}{4} < 2\alpha + \frac{\pi}{4} < \frac{9\pi}{4}$$

$$2\sin^2 x + 3\sin x - 2 = 0$$

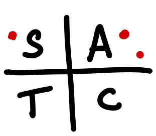
$$(2\sin x - 1)(\sin x + 2) = 0$$

$$2\sin x = 1 \qquad \sin x = -2$$

$$\sin x = \frac{1}{2}$$

$$\sin\left(2\alpha + \frac{\pi}{4}\right) = \frac{1}{2} \quad \text{(or)} \quad \sin\left(2\alpha + \frac{\pi}{4}\right) = -2$$

(reject)



$$2\alpha + \frac{\pi}{4} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$$

$$2\alpha = \frac{-\pi}{12}, \frac{7\pi}{12}, \frac{23\pi}{12}$$

$$\alpha = \frac{-\pi}{24}, \frac{7\pi}{24}, \frac{23\pi}{24}$$

(reject)

13. (a) Show that $\frac{\sin x \tan x}{1-\cos x} = 1 + \sec x$. [4]

$$L.H.S = \frac{\sin^2 x}{\cos x} \times \frac{1}{1-\cos x}$$

$$= \frac{1-\cos^2 x}{\cos x\,(1-\cos x)}$$

$$= \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cos x\,\cancel{(1-\cos x)}}$$

$$= \frac{1}{\cos x} + 1 = 1 + \sec x$$

(shown)

(b) Solve the equation $5\tan x - 3\cot x = 2\sec x$ for $0° \leq x \leq 360°$.

[6]

$$5\frac{\sin x}{\cos x} - 3\frac{\cos x}{\sin x} = \frac{2}{\cos x}$$

$$\frac{5\sin^2 x - 3\cos^2 x}{\cos x \sin x} = \frac{2}{\cos x}$$

$$5\sin^2 x - 3\cos^2 x = 2\sin x$$

$$5\sin^2 x - 3(1 - \sin^2 x) = 2\sin x$$

$$5\sin^2 x - 3 + 3\sin^2 x - 2\sin x = 0$$

$$8\sin^2 x - 2\sin x - 3 = 0$$

$$(4\sin x - 3)(2\sin x + 1) = 0$$

$\sin x = \frac{3}{4}$

$x = \sin^{-1}\left(\frac{3}{4}\right)$

$= 48.6, 131.4$

$\cdot S \mid A \cdot$ / $T \mid C$

$\sin x = -\frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$= 30°$

For negative,

$x = 180° + 30°, 360° - 30°$

$= 210°, 330°$